Simple Noise Analysis Applied to Power Combiners

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Abstract

The advent of ultra-low-noise amplification has shifted the emphasis of low-noise design towards passive circuitry. This paper derives simple expressions for the noise parameters for a passive two-port in terms of the scattering matrix and applies them to the problem of design of power combining circuits. Simple guidelines for low-noise combiner design are presented.

1 Introduction

The availability of ultra-low-noise transistors has shifted the emphasis of low noise design towards concern with passive circuitry. Typical needs for passive circuits include matching and power combining/dividing. These are fabricated using lossy transmission lines, distributed lossy coupling sections, and lumped lossy elements. All of these elements may contribute significantly to the overall noise figure of the network.

Analyses of the noise properties of passive linear circuits using S parameters have been performed before [1, 2]. However, these analyses did not yield a simple procedure for incorporating the results into noise figure calculations. Expressions are derived in section 2 for the standard noise parameters of optimum source match (Γ_{min}), minimum noise figure (F_{min}), and the equivalent noise resistance (R_n). These expressions provide the means for simple incorporation of passive noise into existing CAD software that does not possess this capability.

An application of this formulation to power combining circuits is of interest for the possibility of phased array receivers. The advantages of electrically steerable antennae are obvious but for ultra low noise applications the noise generated in the relatively long transmission lines and balancing resistors may be too high. In section 3, we analyze alternative combiner topologies and propose simple design criteria for low noise combiners.

2 Noise Parameter Derivation

A definition of the noise figure for a two-port is [3]

$$F = \frac{P_{N_o}}{P_{N_i} G_a} \tag{1}$$

where P_{N_o} is the available output noise power, P_{N_i} is the input noise power, and G_a is the available gain. The input and output noise powers can be written in terms of equivalent noise temperature as $P_{N_o} = kT_{out}B$ and $P_{N_i} = kT_{in}B$ where B is the bandwidth and k is Boltzmann's constant. The output noise temperature can be written in terms of the input noise temperature for a passive two-port as

$$T_{out} = T_{in}G_a + (1 - G_a)T_p = T_p + (T_{in} - T_p)G_a \tag{2}$$

where T_p is the physical temperature of the two-port (assumed uniform). Thus the noise figure can now be written as

$$F = \frac{T_p + (T_{in} - T_p)G_a}{T_{in}G_a} = \frac{T_p}{T_{in}G_a} + \frac{T_{in} - T_p}{T_{in}}. \tag{3}$$

To simplify the analysis without loss of generality, assume that $T_p = T_{in}$ (thermodynamic equilibrium). The final solution will only differ from the complete solution by multiplicative and additive constants. Thus the noise figure is simply the intuitive expression

$$F = \frac{1}{G_a} \tag{4}$$

and circles of constant available gain are circles of constant noise figure as well. The available gain is defined as

$$G_a = \frac{|S_{21}|^2 [1 - |\Gamma_s|^2]}{[1 - |\Gamma_{out}|^2] \, |1 - S_{11}\Gamma_s|^2} \tag{5}$$

where Γ_s is the source reflection coefficient and Γ_{out} is the reflection coefficient of the output port with the source connected. Γ_{out} is given by

$$\Gamma_{out} = S_{22} + \frac{S_{21}S_{12}\Gamma_s}{1 - S_{11}\Gamma_s}. \tag{6}$$

Now we have the noise figure in terms of only the S parameters and the source reflection coefficient

$$F = \frac{|1 - S_{11}\Gamma_s|^2 - |S_{22} - \Delta\Gamma_s|^2}{|S_{21}|^2 [1 - |\Gamma_s|^2]} \quad (7)$$

where Δ is the negative of the determinant of the S parameter matrix. Note that for the reflectionless source case the noise figure simplifies to

$$F_0 = \frac{1 - |S_{22}|^2}{|S_{21}|^2}. \quad (8)$$

To find expressions for the noise parameters, we minimize F with respect to Γ_s. This yields the value of source reflection coefficient for minimum noise figure as

$$\Gamma_{opt} = \frac{A - \sqrt{A^2 - 4|C|^2}}{2C} \quad (9)$$

where $A = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$ and $C = S_{11} - \Delta S_{22}^*$. Substituting this into F furnishes the minimum noise figure value

$$F_{min} = F_0 - \frac{|\Gamma_{opt} C|}{|S_{21}|^2}. \quad (10)$$

The last parameter is the equivalent noise resistance. This is obtained by substituting Γ_{opt}, F_{min}, and F into the general expression for two-port noise figure in terms of noise waves [4]

$$F = F_{min} + 4\frac{R_n}{Z_0} \frac{|\Gamma_s - \Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2 [1 - |\Gamma_s|^2]} \quad (11)$$

and solving for R_n. This yields

$$R_n = Z_0 \frac{2Re(C) + A}{4|S_{21}|^2} \quad (12)$$

where Z_0 is the normalizing impedance for the S parameters.

The physical temperature of the two-port may be reintroduced by defining the excess noise temperature ratio

$$T_{ex} = \frac{T_p}{T_{in}} \quad (13)$$

and using equation 3. This yields

$$F_{min}^T = 1 + (F_{min} - 1)T_{ex} \quad (14)$$

and

$$R_n^T = T_{ex} R_n. \quad (15)$$

These equations furnish the noise parameters for all physical temperatures where quantum effects can be ignored.

These formulae may be used for multiports which are symmetric about the input or output port, such as power combiners, by terminating all paths but one in the characteristic impedance to form a two-port. This yields the noise figure of each path multiplied by the number of paths.

3 Verification and Application to Combining Networks

The simplest lossy elements which can be used to verify the above derivation are shunt and series connected resistors treated as two-ports. Equation 10 yields unity for F_{min} in both cases. The optimum source reflection coefficient obtained from equation 9 for the shunt resistor is -1 (i.e. short circuit) and for the series resistor is +1 (i.e. open circuit). The equivalent noise resistance found from equation 12 for the series resistor is R, the actual value of the resistor. The result for the shunt case is equal to zero. Note that in this case the denominator of equation 11 is also zero. The limit for the ratio with $\Gamma_{opt} \to -1$ is also R. These obvious results verify the parameter expressions.

Applying the formulae to a Wilkinson combiner at 4 GHz fabricated in microstrip on an alumina substrate with smooth gold conductors yields the results shown in Figure 1. Analyzing an analogous hybrid 3 dB coupler provides Figure 2.

Note the relative frequency independence of the Wilkinson combiner. Note also how the noise figure of the output port rises monotonically with frequency. The balancing resistor in the Wilkinson contributes nothing to the output noise power as the noise waves induced in the quarter wave lines are completely out of phase for for all frequencies. Use of the Wilkinson as a divider, however, provides noise cancellation of the resistor only at the center frequency, with noise performance degrading rapidly to either side.

The hybrid coupler has a higher noise figure than the Wilkinson and has a strong asymmetry between the two input ports. This is due to the longer line lengths involved and to the dissimilar paths between the 90° and 180° ports.

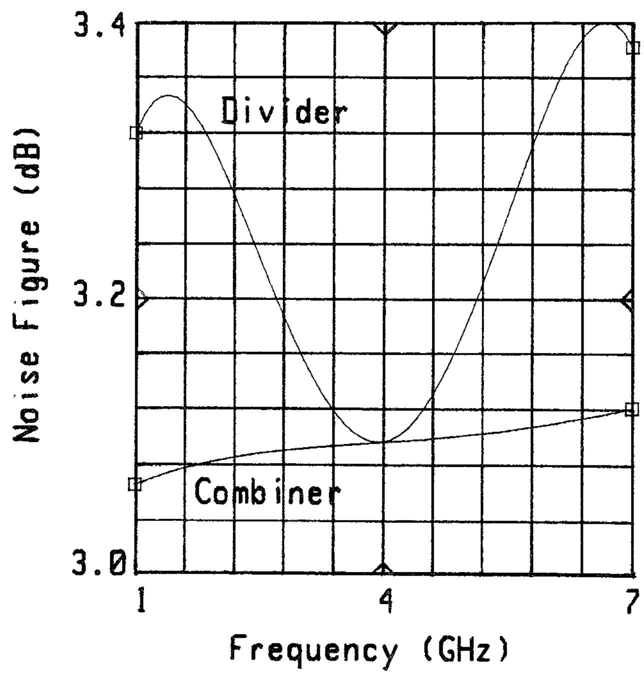

Figure 1: Wilkinson Combiner Noise Analysis.

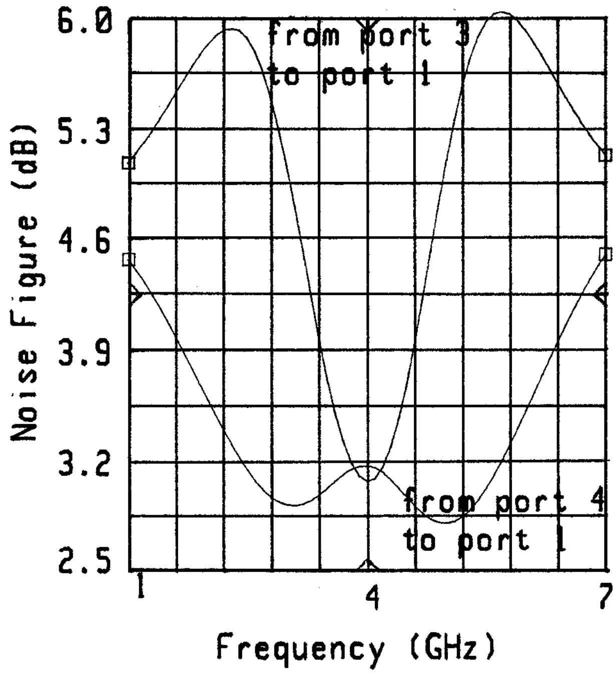

Figure 2: Hybrid Coupler Noise Analysis.

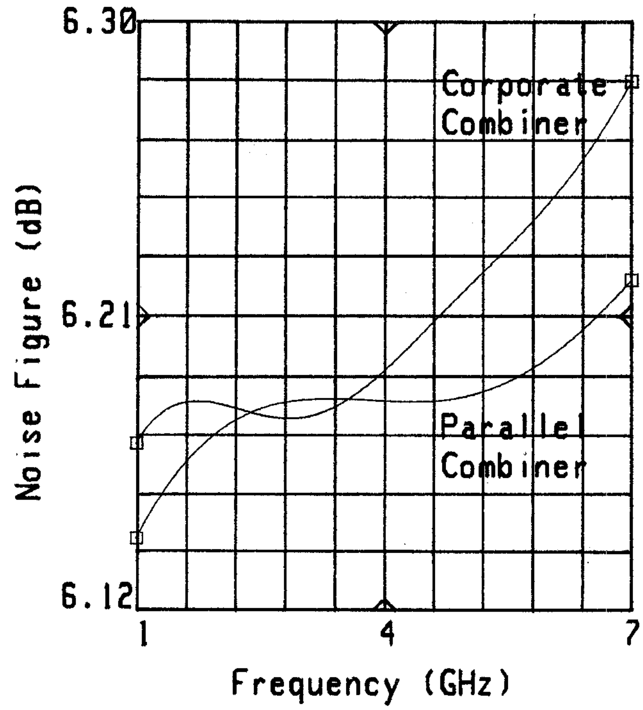

Figure 3: 4-way Wilkinson Noise Analysis

This violates the symmetry criterion used above but illustrates the undesirability of asymmetry.

The noise figures for a 4-way parallel Wilkinson and a 4-way corporate Wilkinson combiner are shown in Figure 3. The corporate combiner has a higher noise figure over most of the band despite the inherent imbalance in a planar multi-way parallel combiner. Thus the series loss proves to be the limiting factor.

4 Conclusion

This new noise figure formulation provides an easily implemented noise analysis of passive two-ports on existing CAD programs that are based on S parameters. Analyses of simple cases verifies the validity of the formulae.

Power combiners were analyzed using this formulation to determine the optimum topology for electrically steerable recievers. The obvious criterium is to minimize the series loss in the transmission lines. A less intuitive result is the total absence of a noise contribution from the balancing resistor in the Wilkinson combiner at all frequencies. This is true even for N-way parallel Wilkinson combiners. However, the noise cancellation applies to dividers only at the center frequency and resistor noise quickly dominates the series loss to either side of midband.

Other combining techniques use more transmission line than the Wilkerson and so will have higher noise figures.

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